

Problem 10) a) Draw a straight-line OEF from O through the center C , crossing the circle at E and F . The triangles OAF and OEB are similar, because they share an angle at O , and also their angles at B and F are identical—both face the arc AE of the circle. The ratio $\overline{OA}/\overline{OE}$ is thus equal to the ratio $\overline{OF}/\overline{OB}$. Consequently, $\overline{OA} \cdot \overline{OB} = \overline{OE} \cdot \overline{OF}$. Since $\overline{OE} \cdot \overline{OF}$ is unique (because the straight-line OEF goes through the center of the circle), the product $\overline{OA} \cdot \overline{OB}$ is the same for *any* straight-line through O that crosses the circle.

b) $\overline{OE} \cdot \overline{OF} = (\overline{OC} - R) \cdot (\overline{OC} + R) = \overline{OC}^2 - R^2$.

c) Considering that CD is perpendicular to the tangent OD , the Pythagoras theorem confirms that $\overline{OD}^2 = \overline{OC}^2 - R^2$.

